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Cofabrication of Electromagnets and Microfluidic Systems in Poly(dimethylsiloxane)**

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Herein we describe a simple method for fabricating electromagnets with micron-scale dimensions in poly(dimethylsiloxane) (PDMS) in close proximity (ca. 10-μm separation) to microfluidic channels. The method has four steps: 1) fabrication of microchannels, which subsequently form both the microfluidic channels and the templates that become the wires for the electromagnets; 2) silanization of the microchannels destined to become wires with 3-mercaptopropyltrimethoxysilane, to make their surfaces wettable by metals; 3) injection of molten solder into the channels; and 4) cooling the channels to form solid metallic wires. These solder wires can be formed into nonplanar structures (if desired), by remelting, shaping the PDMS template, and recooling the solder. By passing electrical current through the wires, we generated magnetic fields up to 2.8 mT, and magnetic field gradients up to 40 $T m^{-1}$, inside adjacent microfluidic channels. To demonstrate the application of the electromagnets in microfluidic systems, we: 1) modeled the magnetic field and Joule heating of an electromagnet; 2) characterized the capture and release of superparamagnetic beads in a microfluidic channel by turning the electromagnets on and off; and 3) sorted superparamagnetic beads into one of two microfluidic channels by applying an electrical signal to a pair of electromagnets (one on each side of a channel).

Magnetic components have the potential to be useful in lab-on-a-chip systems: they form the basis of microfluidic pumps,[1] mixers,[2] and valves,[3] have been integrated into microfluidic systems to trap and move paramagnetic particles,[4,5] and have been used to guide the self-assembly of particles into structures.[6] There are many biochemical applications of magnetic fields: in immunoassays,[7] they have been used to accelerate the hybridization of DNA and RNA,[8] and for the filtration of biomolecules;[9–11] in cell biology, magnets have been used to isolate cells from blood,[12] extract DNA from cells,[13] move magnetotactic bacteria,[14] and to measure the mechanical properties of cells.[15] The use of magnetics in microfluidic systems has been reviewed recently.[16]

Electromagnets have two advantages over permanent magnets in lab-on-a-chip systems: 1) they can be switched on/off rapidly using electrical signals, and 2) the strength of their magnetic field can be adjusted. They have the disadvantage that they usually produce weaker magnetic fields than do permanent magnets. Several groups have fabricated electromagnets in microfluidic systems to manipulate superparamagnetic beads magnetically. Ahn et al. fabricated 3D electromagnets surrounding a microfluidic chamber by electroplating copper wires around a nickel–iron core.[17] Wirix-Speetjens et al. and Smistrup et al. made electromagnets that produced magnetic fields in channels etched in Si/SiO_2.[18,19] Choi et al. fabricated Cu/Ti electrodes in microfluidic channels comprising polyimide, silicon, and glass.[20] Deng and coworkers patterned gold wires with a width of 50–100 μm and thickness of 10–20 μm, and used the wires to produce magnetic fields in PDMS channels.[4] They demonstrated that their wires carried electrical currents as large as 10 A for hours without failure; when the wires were positioned adjacent to a permanent magnet, the wires produced a peak magnetic fields of tens of mT. Lee et al. used lift-off techniques to fabricate gold wires with a width of 1–10 μm and a thickness of approximately 100 nm; an electrical current applied to the wires at a current density $< 5 \times 10^7$ $A cm^{-2}$ produced a magnetic field < 100 mT.[5] Redesigning the wires in a crossbar-array pattern made it possible to trap yeast cells attached to magnetic beads.[21] Suzuki et al.

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micromachined copper wires in silicon to move superparamagnetic beads in a microchannel; applying a current of 1 A produced forces on the beads < 0.3 pN.[22]

While these methods for fabricating electromagnets are useful in some applications, many of them are limited for general use because they require: 1) a bulky off-chip permanent magnet which must be aligned to features on the microchip, or 2) costly and time-consuming deposition of thick (> 1 μm) layers of metal (e.g. Au) to minimize Joule heating in the wires. These techniques require at least two steps of lithography that must be repeated in the fabrication of each device, and also require precise alignment of electrodes to the microfluidic channels.

Herein we describe a method of fabricating electromagnets that does not require an external permanent magnet, or fabrication by evaporation, electrodeposition, or sputtering of metal. It also avoids alignment steps, because the microstructures (e.g. microchannels) that form the wires (that is, the electromagnets) are fabricated in the same plane and at the same time as the microfluidic channels. This cofabrication is essential for practical, low-cost devices, and is part of a strategy we are developing that will allow cofabrication of a range of functions (microfluidic channels, optical waveguides,[23] light sources,[24,25] electrical systems such as wires, electrodes, and heaters[26] and electromagnets) in a single step of molding.

Fabrication procedure: We fabricated electromagnets in microchannels embossed in PDMS, but the general strategy for cofabrication is applicable to many microchannel systems. The PDMS channels were made using soft lithography and rapid prototyping.[27] The interior surfaces of the channels (which had been oxidized by exposure to a plasma prior to contact sealing)[28,29] were made wettable to metal by silanization with a solution of 3-mercaptopropyltrimethoxysilane; liquid solder did not wet untreated channels. We injected molten solder (for example, 52% indium, 48% tin, m.p. 117 °C, or 100% indium, m.p. 158 °C)[30] into the channels by heating them on a hotplate and applying pressure to a heated syringe that was prefilled with the molten solder (Figure 1a); on cooling, the solder solidified into electrically conductive wires. The channels for electromagnets were fabricated in close proximity (ca. 10-μm separation), and in the same plane as the channels used to transport fluid (Figure 1b). By relying on cofabrication, it is straightforward to achieve small separations and excellent registration of microfluidic and electric/magnetic structures. A detailed description of the fabrication procedure is in the Supporting Information.

This method of fabricating electromagnets near microfluidic channels is rapid, simple, reproducible, and requires minimal equipment; silanizing the plasma-oxidized channels, heating, injecting the molten solder into the channels, connecting the copper wires, and cooling is complete in under 10 min. Channels for fluids and electromagnets were cofabricated in the same plane and in a single step; this approach makes it possible to fabricate multiple electromagnets in a microfluidic system using one mask for lithography (no alignment is necessary). Compared to lift-off fabrication using Au, Al, or Cu (metals with slightly higher conductivity and substantially higher melting points than solder), injection of solder makes it possible to fabricate wires with large cross-sectional area rapidly and economically. To generate a given magnetic field, these "thick" wires are

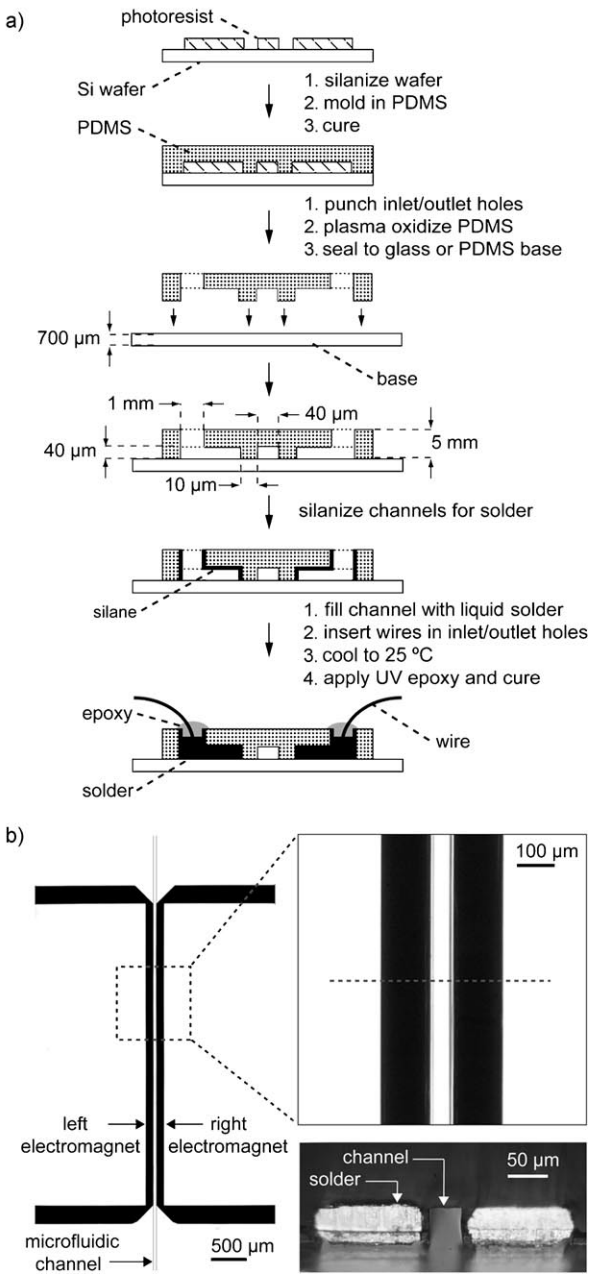

Figure 1. a) A schematic diagram depicting the fabrication of electromagnets in PDMS. The completed device has three microfluidic channels: two outer channels filled with solder (length = 1.5 cm, width = 120 μm, height = 40 μm), and a central channel for fluids (length = 1.5 cm, width = 40 μm, height = 40 μm). b) Photographs of the three channels as viewed from above at low magnification (left), high magnification (upper right) and the cross-section of the three channels (lower right). The photograph of the cross-section was obtained by sectioning the channels with a razor blade (shown as the dashed line in the upper right image); the dark line in the left electromagnet is the result of imperfect sectioning; the light areas at the bottom of the image are reflections of the metal. In the photograph at low magnification, lines were drawn to outline the location of the microfluidic channel.

subjected to lower current density and less heating than smaller wires that generate equivalent fields. We have also been interested in producing complex metallic microstructures in three dimensions;[31,32] in separate work, we have applied this technique to produce flexible 3D structures by bending/twisting/coiling the devices after injecting the solder, or by filling multilayer channels with molten solder and cooling.[26]

We verified that the electromagnets are electrically insulated from the adjacent microfluidic channel. We filled the microfluidic channel shown in Figure 1b with a solution of 0.1M NaCl in Millipore water; the resistance between the adjacent electromagnet and a silver electrode immersed in the solution was 35.5 GΩ. For comparison, the resistance measured between two electrodes immersed in opposite ends of the microfluidic channel filled with the electrolytic solution was 7.6 MΩ, corresponding to a resistivity of 83.4 Ω cm; the resistivity of a solution of 0.1M NaCl in water calculated from the literature is 93.7 Ω cm.[33]

We passed an electrical current through the wires to generate magnetic fields and field gradients in adjacent microfluidic channels; the orientation of these fields and gradients in relation to the direction of the electrical current in the wires is shown in Figure 2. We adjusted the strength of the magnetic field and field gradient by controlling the magnitude of the electrical current through the wires.

Model of the magnetic field: An electrical current passed through a wire generates a magnetic field $\boldsymbol{B}$ (T) around the wire in the direction determined by the right-hand-rule.[34] Equation (1) describes the intensity of this field as a function

$$|\boldsymbol{B}| = \mu_0 \frac{I_{\text{wire}}}{2\pi x} \quad (1)$$

of the distance from the axial center of a cylindrical wire of unlimited length, where I_{wire} (A) is the current through the wire, x (m) is the distance from the center of the wire, and μ_0 ($4\pi \times 10^{-7}$ T m A^{-1}) is the permeability of free space.[35]

Figure 2b shows a plot of the magnitude of the magnetic field produced by passing a constant current through a wire with a rectangular cross section; we approximate this magnetic field using Equation (1). The electromagnets described herein have a cross-sectional area of approximately 4800 μm^2 and can withstand electrical currents >1 A and current densities >22 kA cm^{-2} before Joule heating causes excessive heating of the adjacent fluid; these characteristics make it possible to produce magnetic fields <2.8 mT and field gradients <40 T m^{-1} in microfluidic channels adjacent to the electromagnets (10-μm separation). By comparison, a typical refrigerator magnet produces a magnetic field of approximately 2.5 mT; the strongest permanent rare-earth magnets produce fields up to 1.5 T.[36]

A magnetic field gradient produces a force $\boldsymbol{F}$ (N) on a superparamagnetic bead in the direction of the increase in the magnitude of the field. Lee et al. derived a formula [Eq. (2)]

$$\boldsymbol{F} = \frac{V\chi}{\mu_0} \nabla(\boldsymbol{B}^2) \quad (2)$$

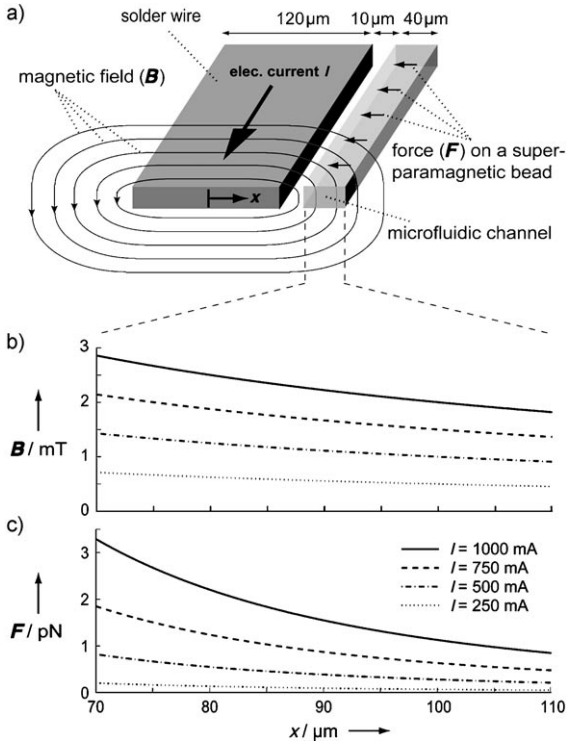

Figure 2. a) A graphical model with corresponding plots of b) the magnetic field (symbols as in (c)), and c) the force exerted on a superparamagnetic bead by an electrical current applied through an electromagnet positioned 10 μm from a microfluidic channel. In the model, the magnetic field is described by field lines orthogonal to the orientation of the solder wire of the electromagnet; horizontal lines in the microfluidic channel describe the force field; x describes the distance from the center of the wire. The microfluidic channel is located between 70 and 110 μm from the center of the wire; the plots are drawn across this range of x. We assume a solder wire of infinite length and uniform current density. In (c), we assume perfectly spherical beads with a susceptibility of 0.170 and a diameter of 5.9 μm. For currents of 250, 500, 700, and 1000 mA, we observed peak magnetic-field intensities (and peak forces) of 0.714 (0.206), 1.43 (0.823), 2.14 (1.85), and 2.86 mT (3.29 pN), respectively.

for calculating this force as function of: 1) the gradient of the square of the magnetic field, $\nabla(\boldsymbol{B}^2)$ (T^2 m^{-1}); 2) the volume of the bead, V (m^3); and 3) the magnetic susceptibility of the bead, χ (dimensionless).[21,37]

Combining Equations (1) and (2) produces an expression for the force exerted on a superparamagnetic bead in the x direction when a direct current (DC) is flowing through a wire [Eq. (3)]. Figure 2c shows a plot of the force as a

$$\boldsymbol{F}_x = -V\chi\mu_0 \frac{I_{\text{wire}}^{\,2}}{2\pi^2 x^3} \quad (3)$$

function of x.

Herein, we calculate a maximum force of approximately 3.3 pN on a superparamagnetic bead (5.9-μm diameter, $\chi = 0.170$).[38] By comparison, typical forces for moving similar superparamagnetic beads range from 0.1–40 pN.[16] Equation (3) suggests that the most effective strategy for making magnetic interactions strong in these systems is to reduce the

distance between the center of the wire and the superparamagnetic bead. We used this strategy to determine the optimal width of the wires used for the electromagnets (see Supporting Information).

Capture and release of superparamagnetic beads: To demonstrate the capture of superparamagnetic beads in microfluidic systems, we fabricated electromagnets in PDMS channels bonded to glass slides using the procedure described in Figure 1a. In these devices, two outer channels were filled with solder to form the electromagnets and the central channel was used for fluids (Figure 1b). We used an electronic circuit to control the electrical current applied to the two electromagnets (see Supporting Information). By turning the electromagnets on either side of the central microfluidic channel on and off, we captured and released superparamagnetic beads[38] from either side of the microfluidic channel at a frequency of 0.3 Hz for over 5000 cycles (one cycle = across the channel and back) (Figure 3a–c). In each cycle, we applied a current of 1 A at a 4 V bias through each electromagnet for 1.6 s. While the electromagnets supported frequencies over 100 kHz, the maximum frequency at which we could observe movement/oscillation of the beads was 50 Hz; this limitation is probably because viscous drag on the magnetic beads prevents them from accelerating quickly in the aqueous media (we give an analysis in the Supporting Information). The maximum frequency at which we could continuously cycle beads across the channel and back was 1 Hz.

We measured the time required to capture a population of superparamagnetic beads[38] in buffer in a microfluidic channel (Figure 3d). We flowed a suspension of beads into a channel, stopped the flow, and imaged a section of the channel that contained 20 beads. We passed a current through an adjacent electromagnet, and recorded the time to "capture" 90% of the beads—that is, to move beads from inside the microfluidic channel into contact with the wall. The "time of capture" was measured at various currents applied to the wire between 130 mA and 1090 mA. The average time to capture the beads was 29 s at 130 mA and 0.75 s at 1090 mA.

The time, t_{cap} (s), required to move a superparamagnetic bead from one sidewall of the microchannel to the opposite wall is approximated by Equation (4); η (10^{-3} kg(m s)$^{-1}$) is

$$t_{cap} = \frac{9}{4}\frac{\pi^2 \eta}{\chi \mu_0 R^2}\frac{(b^4 - a^4)}{{I_{wire}}^2} \quad (4)$$

the dynamic viscosity of the medium, R (m) is the radius of the bead, a (m) is the distance from the center of the electromagnet to the center of a bead resting against the sidewall of the channel, and b (m) is the distance from the center of the electromagnet to the initial position of the bead. A derivation of Equation (4) is in the Supporting Information.

The results of the model are shown as a dashed line in Figure 3d; in the range of 190 mA to 1090 mA, the predicted value of the capture time falls within the range of the measured data. For a current of 130 mA, the predicted value of the capture time is larger than the largest measured value. According to specifications provided by the manufacturer, the beads are weakly magnetized, even in the absence of an applied magnetic field. This factor may explain why the

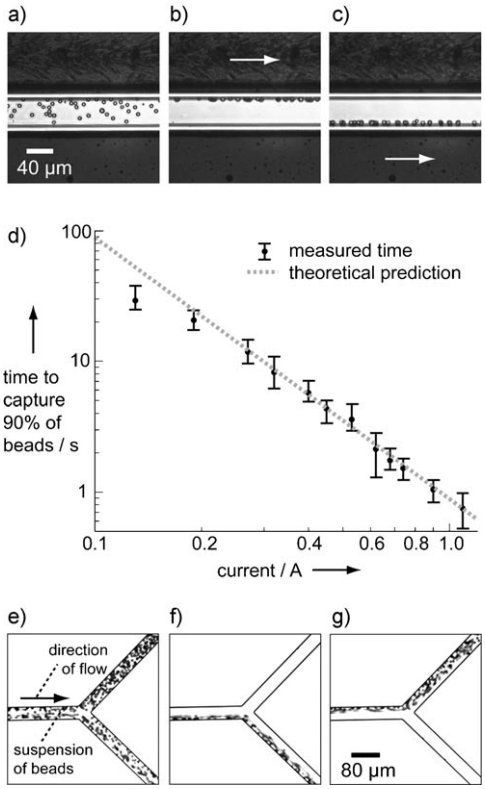

Figure 3. a–c) The capture and release of superparamagnetic beads[38] in a microfluidic channel in proximity to two electromagnets (black). The series of images was taken over 3.2 s. a) The microfluidic channel with no current applied to the electromagnets. b) The microfluidic channel after a current (1 A, in the direction of the arrow) was applied for 1.6 s to the left electromagnet (shown above the channel in the image); the current generated a magnetic field oriented into the plane of the page, with greatest magnitude at the top surface of the microfluidic channel. The gradient in the magnetic field exerted a force on the beads; turning the current on/off made it possible to capture and release beads from the surface of the microfluidic channel. c) The microfluidic channel after the current in the left electromagnet was turned off, and a current of 1 A was applied to the right electromagnet (shown below the channel in the image) for 1.6 s. The process was repeated over 5000 times with similar results. d) Model and measurements of the response time of beads after activation of the electromagnet. Data are the average time to capture (i.e. place in contact with the wall) 90% of a population of superparamagnetic beads measured over 5 experiments; the error bars represent the range of the measured values. The time to capture the beads follows an inverse power relationship with the current [Eq. (4)]. e–g) Sorting superparamagnetic beads in microfluidic channels. Two electromagnets were fabricated on either side of a microfluidic channel 1 cm upstream of a junction; we flowed a suspension of beads through the channel at a rate of approximately 10 μL h^{-1}. e) An image of the junction when both electromagnets were turned off; f) the junction after the right electromagnet was activated for 1 s (and the left electromagnet was turned off); g) the junction after the left electromagnet was activated for 1 s (and the right electromagnet was turned off). For all experiments, the beads were suspended in a buffer at a concentration of 5×10^8 beads mL^{-1} prior to injecting them into the microfluidic channel; the height and width of the channel was 40 μm; the distance between each electromagnet and the channel was 10 μm.

model—which assumes no initial magnetization of the beads—predicts a longer capture time of the beads for small currents, where the relative contribution of the initial magnetization is the strongest.[38]

Sorting superparamagnetic beads: We used the electromagnets to build a switch to sort a stream of superparamagnetic beads[38] flowing in a microfluidic channel into one of two downstream microfluidic channels (Figure 3e–g). When both electromagnets were off, a suspension of beads flowed into both the left and right microfluidic channels at the junction. When the right electromagnet was activated (the left electromagnet was off), the beads were pulled to the right wall of the channel, and were directed into the right microfluidic channel. When the left electromagnet was activated (the right electromagnet was off), the beads were pulled to the left wall of the channel and were directed into the left microfluidic channel. Using an electronic circuit, we alternated the movement of the beads into the left and right channels with a frequency of 0.5 Hz.

In conclusion, the method described herein makes it possible to fabricate metal wires with micron-scale dimensions close to, and in the same plane as, microfluidic channels, while maintaining an insulating barrier between the wires and the channels. The procedure is based on the cofabrication of channels for wires and channels for fluids; this cofabrication dramatically simplifies the process required for fabrication of microdevices. Although the focus is on using 2D wires to make electromagnets, and on their application in manipulating superparamagnetic beads, these structures will also make it possible to produce electrical fields (with strength approaching $10^8\,V\,m^{-1}$) across microfluidic channels by applying an electrical potential between two wires separated by a channel, and to heat fluid in microfluidic channels by running high currents through wires adjacent to the channels. Multiple electromagnets can be constructed in a microfluidic device to produce distinct and independently variable magnetic fields at specific locations in the device (e.g. at distinct locations along the length of a microfluidic channel). Each electromagnet can be activated discretely using independent electrical signals; this capability may be useful for automated capture and release of superparamagnetic beads at specific "stations" in a lab-on-a-chip device.

We believe that this method for cofabricating electromagnets and microfluidic systems will be useful to chemists and biochemists (for manipulating beads functionalized with biomolecules or cells), microsystem engineers (as a new component for microfluidics and integrated function), and applied physicists (as a means of generating electrically controlled magnetic fields in microsystems).

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[1] J. Atencia, D. Beebe, *Lab Chip* **2004**, *4*, 598–602.
[2] L. Lu, K. Ryu, C. Liu, *J. Microelectromech. Syst.* **2002**, *11*, 462–469.
[3] W. C. Jackson, H. D. Tran, M. J. O'Brien, E. Rabinovich, G. P. Lopez, *J. Vac. Sci. Technol. B* **2001**, *19*, 596–599.
[4] T. Deng, G. M. Whitesides, M. Radhakrishnan, G. Zabow, M. Prentiss, *Appl. Phys. Lett.* **2001**, *78*, 1775–1777.
[5] C. S. Lee, H. Lee, R. M. Westervelt, *Appl. Phys. Lett.* **2001**, *79*, 3308–3310.
[6] M. A. Hayes, N. A. Polson, A. A. Garcia, *Langmuir* **2001**, *17*, 2866–2871.
[7] J. Choi, K. W. Oh, J. H. Thomas, W. R. Heineman, H. B. Halsall, J. H. Nevin, A. J. Helmicki, H. T. Henderson, C. H. Ahn, *Lab Chip* **2002**, *2*, 27–30.
[8] H. A. Ferreira, N. Feliciano, D. L. Graham, L. A. Clarke, M. D. Amaral, P. P. Freitas, *Appl. Phys. Lett.* **2005**, *87*, 013901/1–013901/3.
[9] B. L. Hirschbein, D. W. Brown, G. M. Whitesides, *CHEMTECH* **1982**, *12*, 172–179.
[10] S. Ostergaard, G. Blankenstein, H. Dirac, O. Leistiko, *J. Magn. Magn. Mater.* **1999**, *194*, 156–162.
[11] T. Deng, M. Prentiss, G. M. Whitesides, *Appl. Phys. Lett.* **2002**, *80*, 461–463.
[12] I. Safarik, M. Safarikova, *J. Chromatogr. B* **1999**, *722*, 33–53.
[13] S. W. Yeung, I. M. Hsing, *Biosens. Bioelectron.* **2006**, *21*, 989–997.
[14] H. Lee, A. M. Purdon, V. Chu, R. M. Westervelt, *Nano Lett.* **2004**, *4*, 995–998.
[15] N. Wang, D. E. Ingber, *Biochem. Cell Biol.* **1995**, *73*, 327–335.
[16] N. Pamme, *Lab Chip* **2006**, *6*, 24–38.
[17] C. H. Ahn, M. G. Allen, W. Trimmer, Y. Jun, S. Erramilli, *J. Microelectromech. Syst.* **1996**, *5*, 151–158.
[18] R. Wirix-Speetjens, W. Fyen, K. Xu, J. De Boeck, G. Borghs, *IEEE Trans. Magn.* **2005**, *41*, 4128–4133.
[19] K. Smistrup, O. Hansen, H. Bruus, M. F. Hansen, *J. Magn. Magn. Mater.* **2005**, *293*, 597–604.
[20] J. Choi, C. Ahn, S. Bhansali, H. T. Henderson, *Sens. Actuators B* **2000**, *68*, 34–39.
[21] H. Lee, A. M. Purdon, R. M. Westervelt, *Appl. Phys. Lett.* **2004**, *85*, 1063–1065.
[22] H. Suzuki, C. Ho, N. Kasagi, *J. Microelectromech. Syst.* **2004**, *13*, 779–790.
[23] O. J. A. Schueller, X. Zhao, G. M. Whitesides, S. P. Smith, M. Prentiss, *Adv. Mater.* **1999**, *11*, 37–41.
[24] D. V. Vezenov, B. T. Mayers, D. B. Wolfe, G. M. Whitesides, *Appl. Phys. Lett.* **2005**, *86*, 041104/1–041104/3.
[25] B. T. Mayers, D. V. Vezenov, V. I. Vullev, G. M. Whitesides, *Anal. Chem.* **2005**, *77*, 1310–1316.
[26] A. C. Siegel, D. A. Bruzewicz, D. B. Weibel, G. M. Whitesides, submitted..
[27] Y. Xia, G. M. Whitesides, *Angew. Chem.* **1998**, *110*, 568–594; *Angew. Chem. Int. Ed.* **1998**, *37*, 550–575.
[28] D. C. Duffy, J. C. McDonald, O. J. A. Schueller, G. M. Whitesides, *Anal. Chem.* **1998**, *70*, 4974–4984.
[29] A. Y. N. Hui, G. Wang, B. Lin, W. Chan, *Lab Chip* **2005**, *5*, 1173–1177.
[30] Many compositions of solders can be injected into the microfluidic channels to form electromagnets, including: 100% In, m.p. 157°C; 80% In, 15% Pb, 5% Ag, m.p. 148°C; 97% In, 3% Ag, m.p. 146°C; 52% In, 48% Sn, m.p. 118°C. The highest m.p. solder we have used had m.p. 157°C.
[31] S. T. Brittain, O. J. A. Schueller, H. Wu, S. Whitesides, G. M. Whitesides, *J. Phys. Chem. B* **2000**, *104*, 347–350.
[32] H. Wu, S. T. Brittain, J. R. Anderson, B. Grzybowski, S. Whitesides, G. M. Whitesides, *J. Am. Chem. Soc.* **2001**, *123*, 12691–12699.
[33] *CRC Handbook of Chemistry and Physics*, Internet Version (Ed.: D. R. Lide), Taylor and Francis, Boca Raton, **2006**.

[34] M. N. O. Sadiku, *Elements of Electromagnetics*, Saunders, New York, **1989**.
[35] We define the magnetic field in tesla (T), where $1\,T = 1\,N\,(A\,m)^{-1}$.
[36] "About Neodymium Iron Boron" can be found under http://www.mceproducts.com/knowledge-base/article/article-dtl.asp?id=2, **2006**.
[37] Refs. [16], [21], and [22] report slightly different formulas for calculating the force upon a superparamagnetic bead as a function of the gradient of the magnetic field. Our model is based on the formula reported in Ref. [21]; in this model, we neglect any initial magnetization of the superparamagnetic beads and do not consider the inertia of the beads, the contribution of the channel walls to the drag on the beads, or the magnetic susceptibility of the suspending medium.
[38] We purchased COMPEL superparamagnetic beads (5.9-μm diameter) from Bangs Laboratories, Inc. We calculated the magnetic susceptibility of the beads χ using the magnetization curves provided by the manufacturer. The susceptibility ranges from $\chi = 0.174$ at low magnetic fields (0.0–0.5 mT) to $\chi = 0.167$ at high magnetic fields (1.0–3.0 mT); the value of susceptibility that we use ($\chi = 0.170$), is the average value calculated over the general range of applied magnetic fields 0.0–3.0 mT used in our experiments.

